3.2 The derivative

2014-10-06 day 29

Δx means "change in x"

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Δx

[notice that $(x+h) - x = h$, Δx]

Δx

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

inst r.o.c.

$$= \lim_{t_1 \to t_0} \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

3.2 The derivative 2014-10-06 day 29

A function $f(x)$ is differentiable at $x=a$ if the derivative of $f(x)$ at a exists.

i.e. limit exists

A function $f(x)$ is differentiable on an interval (a,b) iff it is differentiable at every x in (a,b)

3.2 The derivative

2014-10-06 day 29

Notation

ways to denote the derivative of $y=f(x)$

$x(t)$

$\dot{f}$ $\dot{x}$

$f'(x)$

$\frac{dy}{dx}$ the derivative of y with respect to x

$f'(x)$ is, itself, a function

$(f')'(x) = \lim_{h\to 0} \frac{f'(x+h)-f'(x)}{h}$

$\Downarrow$

$f''(x)$

"f double prime"

"the second derivative of f with respect to x"

$\frac{d^2y}{dx^2}$: the second deriv of y w.r.t. x

$f'''(x)$

$f^{(4)}(x)$

NOT 4 us.

$\frac{\partial^2 z}{\partial x \partial y}$ the second partial deriv. of z, wrt to x, then wrt y

$z(x,y)$

3.2 The derivative

2014-10-06 day 29

1) A differentiable function is always continuous.

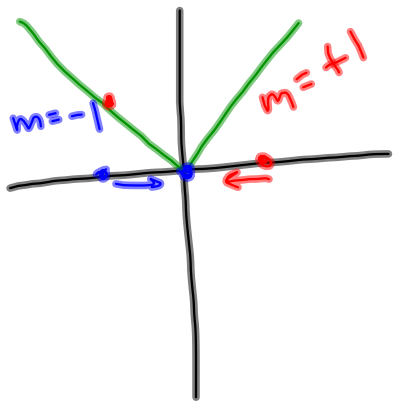
2) ~~A~~ A continuous function is always differentiable.

3) ~~Fns~~ Fns are differentiable AND continuous, or Not differentiable and NOT continuous

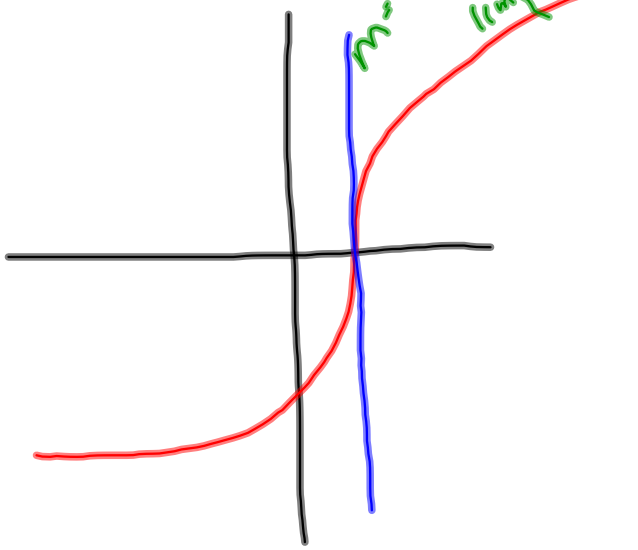
4) no relationship.

Cont fn: not differentiable

1) $|x|$



2) $y = x^{1/3}$



3.2 The derivative

2014-10-06 day 29

A f^n f(x) is continuous at pt a iff

1) f(x) is defined at x=a

2) $\lim_{x \to a} f(x)$ exists

3) $f(a) = \lim_{x \to a} f(x)$